Addition and scalar multiplication are required to satisfy these eight rules,

1. $x+y=y+x$.
2. $x+(y+z)=(x+y)+z$.
3. There is a unique "zero vector" such that $x+0=x$ for all x.
4. For each x there is a unique vector $-x$ such that $x+(-x)=0$.
5. $1x=x$.
6. $(c_1c_2)x=c_1(c_2x)$.
7. $c(x+y)=cx+cy$.
8. $(c_1+c_2)x=c_1x+c_2x$.

Which rule is broken if $cf(x)=f(cx)$? Keep the usual addition $f(x)+g(x)$.

$\cancel{8)}\ (c_1+c_2)x = c_1x+c_2x$

$\underline{(c_1+c_2)f(x)} = c_1f(x)+c_2f(x)$

$f\big((c_1+c_2)x\big) = \underline{f(c_1x)+f(c_2x)}$

$\underline{f(c_1x+c_2x)} =$

Column Space (subspace)

consists of all linear combinations of the columns. The combinations are all possible vectors Ax. They fill the column space $C(A)$.

$$\begin{bmatrix}1\\0\\1\end{bmatrix}a + \begin{bmatrix}0\\2\\1\end{bmatrix}b = \begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}$$

Describe the column space for

$$I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \qquad A = \begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$$

$$Ix = b$$

$$\begin{bmatrix}1\\0\end{bmatrix}x_1 + \begin{bmatrix}0\\1\end{bmatrix}x_2 = \begin{bmatrix}b_1\\b_2\end{bmatrix}$$

$x_1 = 0$, $x_2 = 0$ $\rightarrow \begin{bmatrix}0\\0\end{bmatrix}$ $\Rightarrow$ x-y plane.

$x_1 = 2$, $x_2 = 7$ $\rightarrow \begin{bmatrix}2\\7\end{bmatrix}$

$A=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$

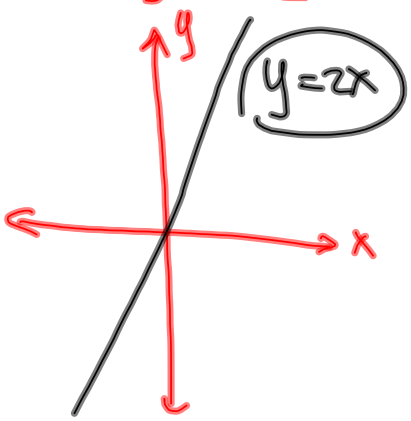
$\begin{bmatrix}1\\ 2\end{bmatrix}x_1+\begin{bmatrix}2\\ 4\end{bmatrix}x_2=$

$\begin{bmatrix}0\\ 0\end{bmatrix}$

$\begin{bmatrix}1\\ 2\end{bmatrix}$

$\begin{bmatrix}2\\ 4\end{bmatrix}$

$\begin{bmatrix}28k\\ 56k\end{bmatrix}$



$B=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$

Solve.

$$\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}-10\,x_1\ 1\\ 8\,x_2\ 3\end{bmatrix}=\begin{bmatrix}7\\ 11\end{bmatrix}$$

$\begin{bmatrix}6\\ 4\end{bmatrix}$

$\begin{bmatrix}a\\ b\end{bmatrix}$